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Estimation of Aircraft Structural Fatigue Life Using the Crack Severity Index Methodology

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In this work, fatigue test results under different random spectra have been used to validate the crack severity index methodology in which only the normal load factor and aircraft gross weight were considered as the load. The original crack severity index methodology was developed to quantify the relative severity in terms of crack growth potential using the stress spectra recorded at the wing root. The crack severity index values have been used to determine the flight mission severity through interpolation between known flaw growth curves of reference usages. This interpolated flaw growth curve enables aircraft users to predict the cumulative fatigue crack growth damage based on individual aircraft activity and provides a useful guide for scheduling future inspection and maintenance activities. To validate the crack severity index methodology based on normal load factor and aircraft gross weight data instead of stress spectra, experimental fatigue lives were obtained for the stresses represented by normal load factor and aircraft gross weight spectra of different random loads. Then, the differences in fatigue life as severity measurements were compared with the crack severity index values. A good correlation was obtained, which validated the crack severity index methodology. The test results were also used to tune a simulation equation for crack growth. The good correlation between the crack severity index values and fatigue test or crack growth simulation results suggests a simple way of predicting the structural fatigue life from crack severity index values.

Nomenclature

a	=	crack length
C	=	normalization constant
m	=	material constant
N	=	applied fatigue cycles
n	=	empirically derived constant
R	=	stress ration
$S_{\max}/\sigma_o$	=	ratio of the maximum applied stress to the flow stress
α	=	plane stress/strain constant
$\beta(a)$	=	geometry function
Δa	=	crack increment
ΔK	=	stress intensity factor
ΔK_{eff}	=	effective stress intensity factor
$\Delta\sigma_{\mathrm{eff}}$	=	effective stress range
$\sigma_{\max}$	=	maximum stress
σ_{op}	=	crack opening stress

I. Introduction

THE structural fatigue life of an aircraft depends not only on the operational environments but also on the mission severity of each aircraft. Thus, the variations in the mission severity experienced by the aircraft throughout its life and the need to identify the operational loads have made the individual aircraft tracking (IAT) programs necessary. Furthermore, to assess the consumed fatigue life of an aircraft structure, the information of the actual load experienced by that structure is essential [1]. Modern military aircraft, therefore, are equipped with flight data recorders to monitor operational loads individually. The calculated rate of fatigue damage accumulation from individual operational loads using IAT program may be higher or lower than the reference rate due to operational variations. In fact, some aircraft are retired at different number of flight hours because of various fatigue damages although all have the same design life.

The crack severity index (CSI) is one of the methodologies implemented for modern military aircrafts such as F-16 aircraft to define mission severity relative to a known reference usage. The CSI developed by National Aerospace Laboratory (NLR) in the Netherlands can be used for quantification of the fatigue damage of recorded stress spectra in terms of crack growth potential [2]. An important quantity is the minimum crack opening stress based on crack closure and crack growth retardation. This opening stress is related to load interaction effects between large and small load cycles. The fatigue damage of a flight is, therefore, dependent on the severity of the flights which has been flown before that flight [3].

The NLR approach uses a maneuver stress history obtained from strain measurements recorded in the wing root area by the Spectrapot data recorder, while the Lockheed Martin Aero implementation of CSI uses normal load factor (Nz) and aircraft gross weight (W) data [4]. The stress based CSI is fatigue related, while calculating CSI from the NzW data recorded in the crash survival flight data recorder (CSFDR) is only to define maneuver severity relative to a known reference environment rather, not to predict fatigue life. For an estimation of fatigue life, the CSI values are used to interpolate between known crack growth curves representing a reference usage and flight mission severity. But an interpolated fatigue life can be over or under estimated. Therefore, it is desirable to correlate the CSI

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values obtained from NzW data with fatigue test results with respect to different severity of NzW spectra thereby quantify the validity of NzW based CSI methodology.

In this work, tests to estimate fatigue life were carried out on compact tension specimen for NzW time history data of different severities downloaded from CSFDR. Simultaneously the CSI values were calculated from the ΔNzW from–to occurrence tables extracted from the same NzW time history data through several data manipulating procedures. Finally, the calculated CSI values are correlated with fatigue test results. The test results were also used to find the NASGRO equation constant to simulate the fatigue characteristics. The constant was found to be linearly related to CSI values. Thus, with the correlation and/or the simulation equation, CSI can provide estimation of fatigue life associated with a mission severity.

II. Development of Load Spectrum

Several kinds of flight data were downloaded from the CSFDR of the F-16 aircraft to develop load spectra of normal load factor and aircraft gross weight. Three sets of flight data among them were selected as mild, normal, and severe loading conditions representing overall flight status in terms of the relative severity of aircraft missions. Only Nz (normal acceleration) and W (aircraft gross weight) data were extracted from time history flight data to represent three different severities of 500 flight hour block spectra. To be accepted as random load spectra, the raw data must be filtered to remove invalid data, searched to extract only peaks and valleys, and then tabulated by rain-flow counting algorithm. Also, the filtered time history data were transformed into the format of from–to occurrence table. Figure 1 shows NzW time history data of three different severities of 500 flight hours. Table 1 is the summary of 500 h time history data. The plots of cyclic peak ΔNzW exceedances per 1000 flight hours obtained from from–to occurrence table may be used as a useful tool to compare relative severities of different spectra as shown in Fig. 2. For the purpose of fatigue analysis and test, NzW time history data must be converted into stress spectra using the transfer function between NzW and wing root stress of the F-16 aircraft. In this work, the wing root stress is treated as directly proportional to NzW.

III. Calculation of CSI Values

The main purpose of the Crack Severity Index is to describe the severity of load spectra relative to the reference (mission) spectra. The CSI methodology originally used stress spectra in a maneuver history obtained from strain measurements in the wing root area by the spectrapot data recorder. However, in this study, NzW time history data obtained from CSFDR were used directly as the stress data, as done by Lockheed Martin.

As shown in Fig. 3, NzW time history data downloaded from CSFDR were converted into random NzW spectra through several data manipulating procedures such as peak and valley search

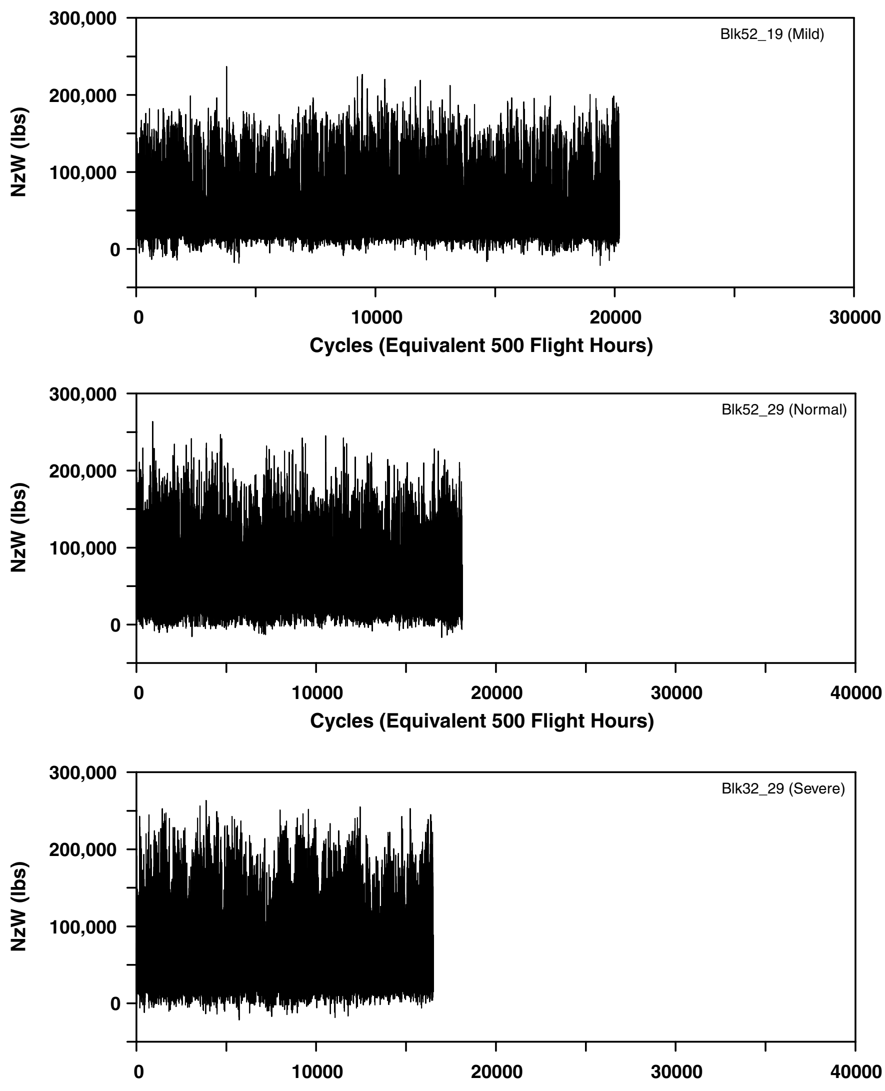

Fig. 1 Three different NzW time history spectra.

Table 1 Summary of 500 Hours NzW time history data

	Mild	Normal	Severe
Total cycles	6401	8952	9414
Maximum value, lb	248,064	263,520	271,392
Minimum value, lb	−21, 504	−16, 560	−21, 536

algorithm and rain-flow counting method. Also, NzW data stored in the form of ΔNzW from–to occurrence table in the CSFDR was used to calculate CSI value for different severity spectra.

It is assumed that the stress cycle spectrum is the result of a rain-flow analysis representing the successive peaks and valleys in the measured stress history. In the from–to matrix as shown in Fig. 4, the physical stress can be described by means of from–to matrix $[i, \quad j]$, which is the number of stress changes from level j to i. As indicated, the numbers below the main diagonal show the upward change of stress, while the numbers above the main diagonal show the downward change of stress. For the development of CSI, it is assumed that crack growth occurs in positive or upward stress change and that crack increment Δa is associated with the effective stress intensity range at the crack tip. The first fatigue crack growth law as a function of the effective stress intensity factor at the crack tip was proposed by Paris and Erdogan [5]

$$\Delta a = C(\Delta K_{\text{eff}})^m = C[\beta(a)\sqrt{\pi a}]^m(\Delta\sigma_{\text{eff}})^m \tag{1}$$

where a is the crack length, $\beta(a)$ a geometry function, and $\boldsymbol{m}$ a material constant. The effective stress range can be written as maximum stress and crack opening stress

$$\Delta\sigma_{\text{eff}} = \sigma_{\max} - \sigma_{\text{op}} \tag{2}$$

Spectrum crack growth equation can be rewritten from Eq. (1) by defining crack geometry function $F(a)$. In this equation, the left-hand side is defined by the structural geometry, crack length, and material properties. The right-hand side includes stress spectrum term and the material parameter

$$\frac{\Delta a}{[\beta(a)\sqrt{\pi a}]^m} = C(\Delta\sigma_{\text{eff}})^m, \qquad F(a) = \int \frac{\mathrm{d}a}{[\beta(a)\sqrt{\pi a}]^m} \tag{3}$$

By integration of Eq. (3) from a_e to a_b, the crack growth can be expressed as in Eq. (4) and it defines the crack severity index. Hence, the right-hand side of Eq. (5) defines the severity of a spectrum related to crack growth and the CSI is the summation of effective stress

$$F(a_e) - F(a_b) = C\sum_i (\Delta\sigma_{\text{eff},i})^m \tag{4}$$

$$\text{CSI} = C\sum_i (\Delta\sigma_{\text{eff},i})^m \tag{5}$$

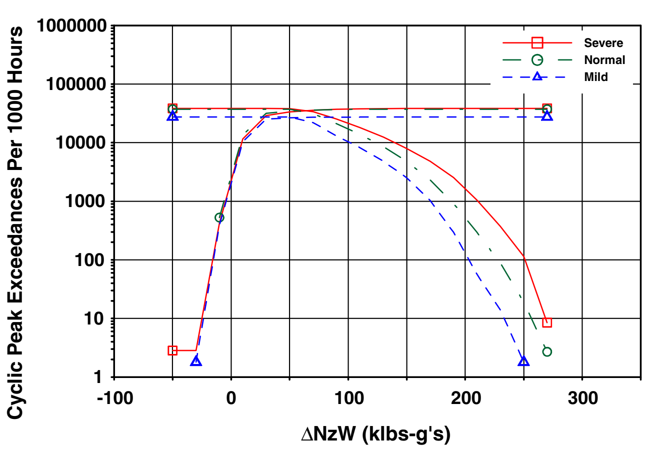

Fig. 2 Comparison of ΔNzW exceedances in three spectra.

$$\begin{aligned} & R = \sigma_{\min}/\sigma_{\max} = 0 \qquad R \leq -4.166 \\ & \sigma_{\text{op}}/\sigma_{\max} = 0.25 + 0.06R \\ & -4.166 < R \leq 0 = 0.25 + 0.06R + 1.13R^2 - 0.44R^3 \\ & 0 < R \leq 1 \end{aligned} \tag{6}$$

The procedure to calculate the CSI is as follows. The first step is to determine crack opening stress level to derive the effective stress level in Eq. (5). Without consideration of retardation effect, the crack opening stress can be approximated by Eq. (6) with stress ratio. In the approximated retardation model, crack opening stress level can be defined by Eq. (6) or equal to minimum opening stress, whichever is the largest [6]. The minimum opening stress is a function of the shape of the load spectrum considered. In this study, the minimum opening stress was calculated from Eq. (6) by substituting the stress ratio of the highest stress to the lowest stress in the time history data.

The second step is to derive the material constant $\boldsymbol{m}$. According to [6], $\boldsymbol{m}$ value has a wide range depending on the type of material and type of load spectrum. The $\boldsymbol{m}$ value in this study was set as three identical to that of NLR.

The last procedure is to calculate normalization constant $\boldsymbol{C}$ of Eq. (5). It is not necessary, however, to calculate normalization constant $\boldsymbol{C}$ because the crack growth potential obtained from the normal spectrum is a reference value. In other words, once the crack growth potential values are calculated from different severity spectra and then the values are divided by normal crack growth potential value, the constant $\boldsymbol{C}$ drops out and the CSI values are obtained automatically. For the NzW-based CSI methodology, σ in Eq. (5) is replaced by NzW as the stress is taken to be directly proportional to NzW.

Table 2 shows the CSI results calculated from NzW from-to table data for three different severities of spectra. In this table each threshold potential is equivalent to the right side of Eq. (5) and normalized occurrences are total numbers of each ΔNzW range per 1000 flight hours. According to ΔNzW range from 0 to 260, specific damages can be calculated by multiplying threshold potential and normalized occurrences together. Crack growth potential (CGP) is the summation of specific damages coincident with ΔNzW range. Finally, CSI value was calculated as the ratio of the CGP for mild, normal, and severe spectrum to the CGP of normal spectrum.

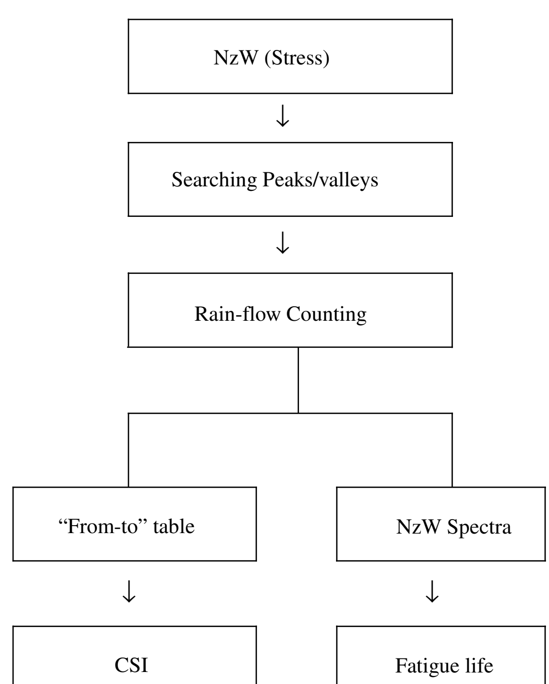

Fig. 3 The procedure to develop NzW spectra and CSI value.

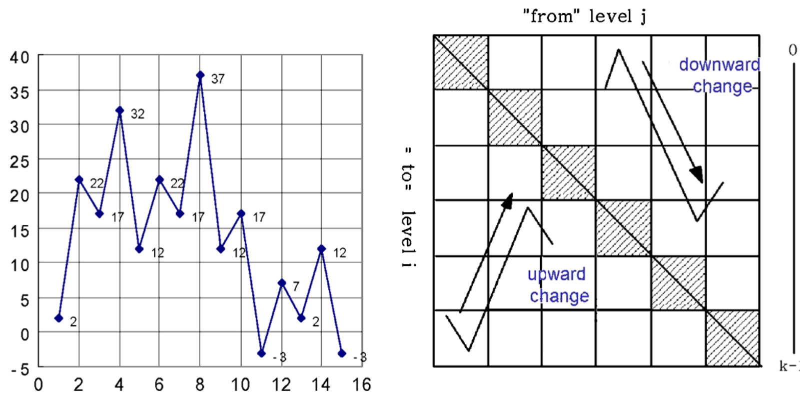

Fig. 4 The stress cycle format and from–to matrix.

IV. Fatigue Experimental Setup and Test Results

The fatigue crack growth test was performed using MTS 810 dynamic fatigue machine to estimate the structural fatigue life under different severities of NzW spectra. The compact tension (CT) specimen made of Al 7075T6 alloy, 2 in. in width (W) and 0.19 in. in thickness (B) as shown in Fig. 5, was prepared to the ASTM E647 specification [7]. The notch of 0.07 in. in length was made by precision diamond saw to prompt crack initiation. To consider only the fatigue crack growth life excluding crack initiation time, the fatigue crack was initiated and propagated until initial crack length a reached 0.5 in. under constant amplitude loading. Once the crack reached 0.5 in length, NzW load spectra were applied to propagate fatigue crack and then crack length from the initial crack was measured by in-situ monitoring method. Normalized NzW spectra by the maximum value of NzW, were used as input load profiles for MTS machine. Scale factor was set as 1000 times considering the geometry and fracture properties of CT specimen. Normalized NzW time history data were converted into the specific format of data file as an external input for MTS machine to simulate NzW load spectrum. Figure 6 shows the experimental setup for fatigue crack growth test using MTS fatigue machine equipped with crack monitoring system. Crack length was measured using CCD camera equipped with infinite focal length lens during the test in-situ.

The fatigue crack growth tests were repeated more than 3 times for each NzW spectrum. Figure 7 shows the fatigue test results representing average and scattered values of fatigue crack lengths for mild, normal, and severe load spectra. Because CT standard specimen rather than coupon to represent structural parts of control points was used for tests, absolute fatigue life was not represented in terms of flight hours. Rather, from the point of relative fatigue life, the test results show reasonable trend; the mild spectrum indicating the longer fatigue life, while the severe one indicating the shorter fatigue life. With the normal load spectrum as the base, the fatigue life under severe spectrum is reduced to one half and under the mild case it is increased to about twice. Also, the test results show that the more severe the load spectrum, the wider the scattering of the data. However, the scattering range is still relatively narrow considering the widely varying nature of fatigue life in general, showing relatively good reliability or repeatability of the test results for the purpose of comparing the relative fatigue life.

V. Comparison of CSI Results with Fatigue Life

The CSI values calculated from the NzW from–to table data were compared with the fatigue test results presented above to quantify the CSI against the severity index based on fatigue life. Each fatigue life was inversed to convert to a value representative of severity comparable to CSI value, which is called relative inverse fatigue life. The NzW based CSI values and relative inverse fatigue lives are also compared with those of NLR [6]. As mentioned in the previous section, the NLR results were based on strain data measured at wing root area. For the comparison, the mild and severe cases are normalized to those of reference or normal loadings, forming the severity index ratios. As shown in Fig. 8, results show good correlations between CSI values and inverse fatigue lives regardless whether the stress spectra used for estimation of relative severity are the measured wing stresses, NzW from–to or the fatigue tests. The CSI value under severe spectrum was underestimated by about 20% of the fatigue life, while in case of mild spectrum the CSI was overestimated by nearly 20%.

Table 2 CSI results with respect to mild, normal, and severe NzW spectra

ΔNzW Range	Mild			Normal			Severe		
	Threshold potential	Normalized occurrences	Specific damages	Threshold potential	Normalized occurrences	Specific damages	Threshold potential	Normalized occurrences	Specific damages
0	0.00	66	0	0.00	102	0	0.00	58	0
20	1.21	2493	3022	1.30	3000	3893	1.75	2212	3873
40	12.12	9328	113,045	12.09	11110	134,310	20.44	11340	231,744
60	57.27	5604	320,964	52.43	8302	435,244	83.07	9024	749,637
80	190.58	2960	564,173	166.98	5348	892,994	220.65	5494	1,212,262
100	383.65	2014	772,635	365.19	3420	1,248,952	461.34	3830	1,766,915
120	714.89	1386	990,607	703.73	2268	1,596,060	882.60	2642	2,331,833
140	1151.41	875	1,007,200	1201.89	1390	1,670,631	1395.93	1570	2,191,606
160	1755.65	453	795,802	1753.40	660	1,157,241	2043.27	1068	2,182,214
180	2580.95	147	379,703	2755.84	332	914,940	3016.53	526	1,586,695
200	3994.67	32	127,067	3769.28	114	429,698	3902.37	238	928,764
220	5424.82	6	32,355	5466.56	16	87,465	5262.21	98	515,696
240	0.00	0	0	6887.08	10	68,871	5989.84	28	167,715
260	0.00	0	0	0.00	0	0	8331.35	8	66,651
	Crack growth potential		5,106,572	Crack growth potential		8,640,299	Crack growth potential		13,935,606
	CSI		0.59	CSI		1	CSI		1.61

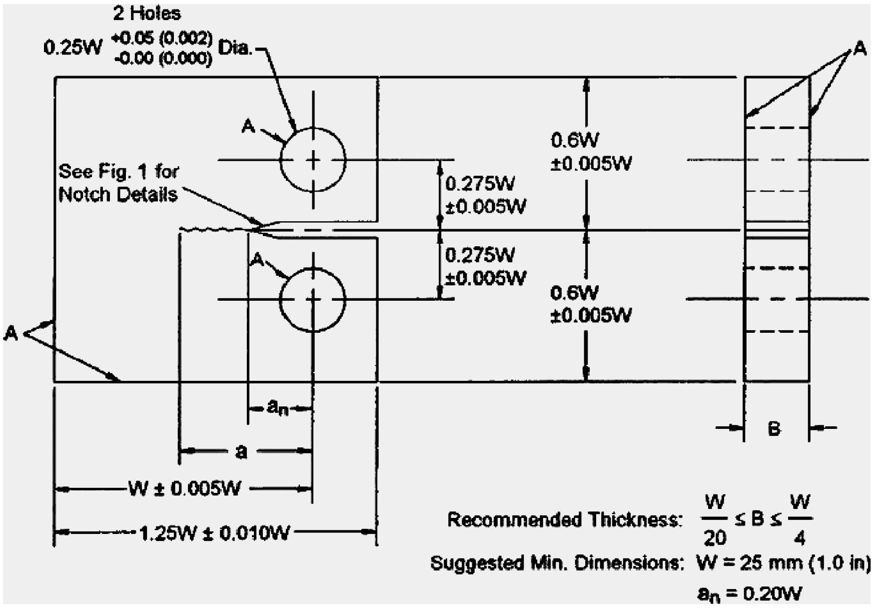

Fig. 5 The geometry of compact tension specimen.

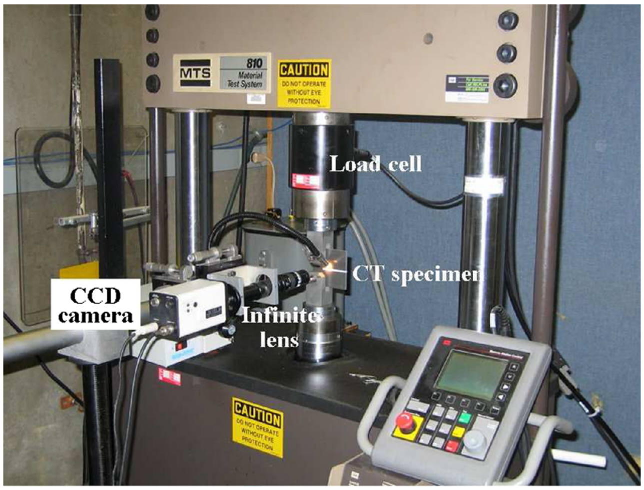

Fig. 6 Experimental setup for fatigue test.

Therefore, it can be concluded that the CSI methodology using NzW spectra can be also used to estimate relative fatigue life for different mission severity. However, it is noted that the CSI value is only able to predict relative fatigue life in terms of ratio to the baseline fatigue life rather than absolute fatigue life regardless of reference.

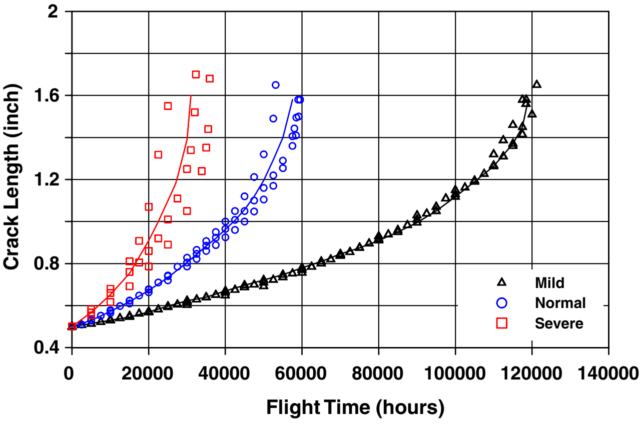

Fig. 7 Fatigue crack growth curve with respect to NzW load spectra.

Reliable estimation of fatigue crack propagation and remaining life prediction are essential for structural integrity assessment based on damage tolerance considerations [8]. Fatigue crack growth analysis is to calculate fatigue crack propagation of cyclically loaded structures that contain initial cracklike defects. In this work, NASGRO module in the MSC Fatigue commercial software was used to analyze fatigue crack growth. Crack growth rate calculations in NASGRO module use a relationship called NASGRO equation [9]. It is given by Eq. (7)

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\left[\left(\frac{1-f}{1-R}\right)\Delta K\right]^n \frac{(1-\frac{\Delta K_{\mathrm{th}}}{\Delta K})^p}{(1-\frac{K_{\max}}{K_c})^q} \tag{7}$$

where N is the number of applied fatigue cycles, a is the crack length, R is the stress ratio, ΔK is the stress intensity factor range, and C, n, p, and q are empirically derived constants. The crack opening function f for plasticity-induced crack closure has been defined by Newman [10] as

$$f = \frac{K_{\mathrm{op}}}{K_{\max}} = \begin{cases} \max(R, A_0 + A_1 R + A_2 R^2 + A_3 R^3) & R \geq 0 \\ A_0 + A_1 R & -2 \leq R < 0 \end{cases} \tag{8}$$

and the coefficients are given by

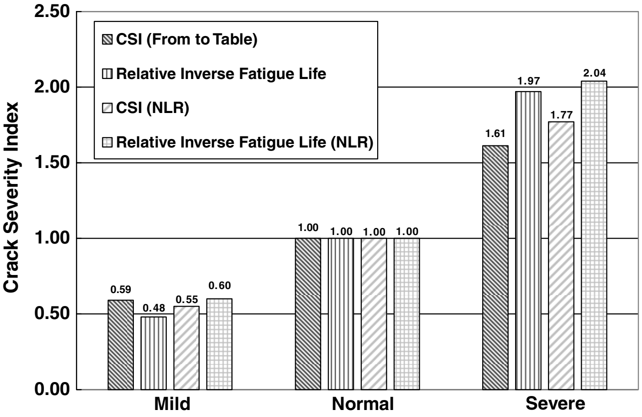

Fig. 8 Comparison of CSI results with fatigue life.

$$A_0 = (0.825 - 0.34\alpha + 0.05\alpha^2)\left[\cos\left(\frac{\pi}{2}S_{\max}/\sigma_0\right)\right]^{1/\alpha} \quad (9)$$

$$A_1 = (0.415 - 0.071\alpha)S_{\max}/\sigma_0 \quad (10)$$

$$A_2 = 1 - A_0 - A_1 - A_3 \quad (11)$$

$$A_3 = 2A_0 + A_1 - 1 \quad (12)$$

In the above equations, α is a three dimensional constraint and plane stress or plane strain conditions are simulated in the model with $\alpha = 1$ or 3, respectively. $S_{\max}/\sigma_0$ is the ratio of the maximum applied stress to the flow stress. The flow stress σ_0 is taken to be the average between the uniaxial yield stress and uniaxial ultimate tensile strength of the material.

To compare fatigue crack growth analysis with test results three different severities of NzW load spectra were converted into predefined forms of input data as the applied loads. All material properties for Al 7075T6 were selected from the NASGRO material libraries. Empirically derived constant n was changed repeatedly until the difference between analysis and test results is sufficiently reduced or eliminated. Other NASGRO equation constants C, p, and q were automatically assigned by the program as default. By tuning the n between 2.35 and 2.65 the crack growth is very well matched with the experimental results as shown in Fig. 9. As the severity of spectrum is increased, the best fit of n per each spectrum is also slightly increased; mild 2.35, normal 2.45, and severe 2.65. In fact, n is linearly related as shown in Fig. 10. Thus, the unified crack growth model to predict fatigue life under wide range of load spectra could not be obtained in this study. Since each New load spectrum used for this work has different amplitude profiles and different cycles of block loading equivalent to 500 flight hours as explained in Table 1, it is not easy for unified crack growth equation to predict fatigue life well in terms of flight hours rather than flight cycles. However, there is a linear relationship between crack severity index and NASGRO equation constant, n, as shown in Fig. 10. When such relationship is available, it can be used to obtain the NASGRO constant n for crack growth simulation for a known CSI value without experimental verification.

VI. Discussion

Predicting the fatigue crack growth under variable amplitude loading is a difficult but significant task in the analysis of damage tolerance [11]. As one of the main tasks in the Aircraft Structural Integrity Program, development of load spectra showing flight maneuver severity is a prerequisite for damage tolerance analysis. However, before the 1990s making acceptable load spectra from the downloaded flight data was almost impossible due to the limited memory in the flight data recorders used for the Individual Aircraft Tracking program. Therefore, instead of taking load spectra at regular intervals, an extrapolation method to predict fatigue life indirectly from the reference fatigue life was widely used. The CSI methodology is one of the typical indirect fatigue life monitoring methods. On the other hand, in the current system of modern aircraft it is possible from downloaded flight data to make load spectra regularly for IAT program as the memory size of flight data recorders has increased. These load spectra have been derived directly from flight data recorders using the relationship between external loads and flight parameters. Predicting fatigue crack growth rate under given load spectra requires experimental guidance and verification for damage tolerance analysis. However, verification of predicted fatigue life through experimental results is a highly time consuming effort. The use of the relationship between the NASGRO constant n CSI offers a way of avoiding the tedious fatigue testing.

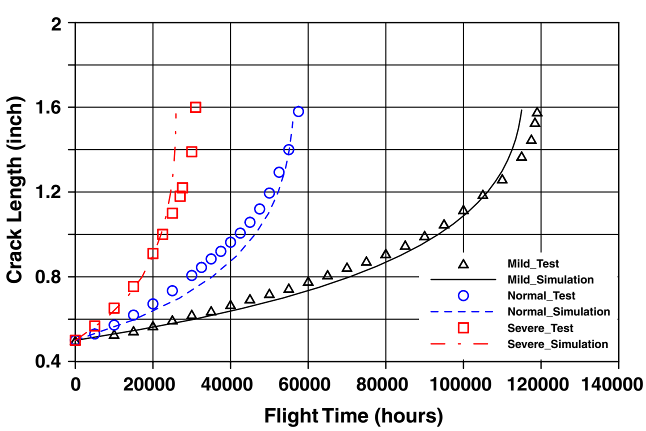

Fig. 9 Correlation between fatigue crack growth simulation and fatigue test results with respect to NzW load spectra.

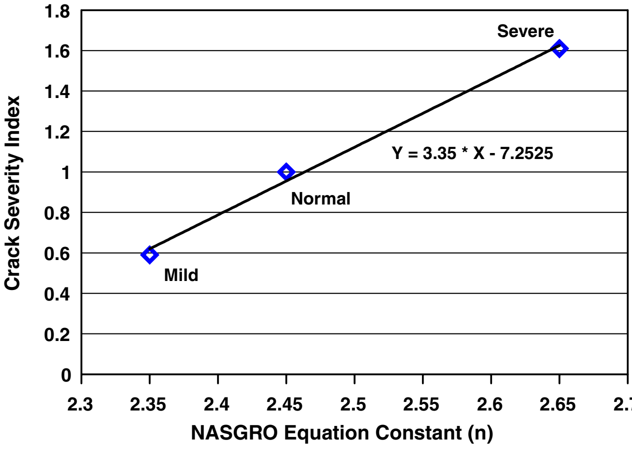

Fig. 10 Relationship between CSI and NASGRO constant.

VII. Conclusions

In this study, the use of NzW based CSI methodology for estimation of fatigue life was investigated. Fatigue tests using NzW data as the load spectra representing three different mission severities were conducted and inverse fatigue life was formed from each resulting fatigue life. The CSI and inverse fatigue life for the normal mission severity were used as the references. Then, the percentage differences in inverse fatigue life relative to the reference were compared with corresponding differences in the CSI. Reasonably good correlation was obtained.

Thus, it is shown that the NzW based CSI methodology provides reasonably good estimation for relative fatigue life. However, because the CSI value is only able to predict relative fatigue life in terms of baseline fatigue life, the absolute value of fatigue life can be estimated only when the reference fatigue life is known. Therefore, when the variation in usage causes changes in the load spectrum for the reference fatigue life, the procedures to calculate CSI value should be updated based on new spectrum.

The test results were also used to determine the NASGRO equation constants. Fatigue crack growth obtained using NASGRO software is well matched with experimental results by tuning the NASGRO equation constant, n. The best fit of n per each spectrum showed slightly increasing trend with the severity of spectrum, indicating a unified crack growth equation to predict fatigue life in terms of flight hours can not be obtained. However, the relationship between CSI and the NAGRO constant n can be used to simulate crack growth by the NASGRO equation.

Thus, the CSI methodology is useful not only in predicting relative fatigue life based upon the reference but also in tuning the NASGRO equation without experimental verification so that fatigue crack growth can be simulated for the purpose of damage tolerance analysis.

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